Sampling Distributions

1. Random Sample:

A collection of independent and identically distributed (iid) r.v. from some distribution is its random sample.

2. IID:

Each r.v. has the same probability distribution and are mutually independent. However, they do not need to have the same probability.

3. Statistic:

Any function of a random sample. The distribution of a statistic is called the sampling distribution of that statistic.

4. Sample Mean:

A sample is a part of the population. For example, a polling company won't ask every Canadian citizen a question, but will ask a small amount. That small amount is the sample.

There are 2 ways to denote sample mean, M_n and $\overline{X}$. If the sample size is fixed, use $\overline{X}$. Otherwise, use M_n.

Formula:

$$\overline{X} = M_n = \frac{X_1 + X_2 + \dots + X_n}{n}$$

Convergence in Probability

1. Let $X_1, X_2, \ldots$ be an infinite seq of r.v. and let y be another r.v. Then, the seq $\{X_n\}$ converges in probability to y if for all $\varepsilon \geq 0$,
$$\lim_{n \to \infty} P(|X_n - y| \geq \varepsilon) = 0.$$

We write this as $X_n \xrightarrow{P} y$.

E.g. 1

Let $Y \sim$ Uniform $[0,1]$ and let $X_n = y^n$. Prove that $X_n \to 0$ in probability.

$$\lim_{n \to \infty} P(|X_n - 0| \geq \varepsilon)$$

$$= \lim_{n \to \infty} P(y^n \geq \varepsilon)$$

$$= \lim_{n \to \infty} P(y \geq \varepsilon^{\frac{1}{n}})$$

$$= 1 - \lim_{n \to \infty} P(y \leq \varepsilon^{\frac{1}{n}})$$

$$= 1 - \lim_{n \to \infty} \int_0^{\varepsilon^{\frac{1}{n}}} 1 \, dt$$

$$= 1 - \lim_{n \to \infty} \left[t \Big|_0^{\varepsilon^{\frac{1}{n}}} \right]$$

$$= 1 - \lim_{n \to \infty} e^{\frac{1}{n}}$$

$$= 1 - 1$$

$$= 0$$

$$\therefore X_n \to 0$$

2 The Weak Law of Large Numbers:

Let $X_1, X_2, \ldots$ be a sequence of indep r.v. each with the same mean, $E(X_i)$. Then, for all $\varepsilon > 0$,

$$\lim_{n \to \infty} P(|M_n - E(X_i)| \geq \varepsilon) = 0.$$

I.e. This means $M_n \xrightarrow{P} E(X_i)$.

The alternative definition is $|M_n - E(X_i)| < \varepsilon$ approaches 1 as n approaches infinity.